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First-Order Electroweak Phase Transition in the Standard Model with a Low Cutoff

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We study the possibility of a first-order electroweak phase transition (EWPT) due to a dimension-six operator in the effective Higgs potential. In contrast with previous attempts to make the EWPT strongly first-order as required by electroweak baryogenesis, we do not rely on large one-loop thermally generated cubic Higgs interactions. Instead, we augment the Standard Model (SM) effective theory with a dimension-six Higgs operator. This addition enables a strong first-order phase transition to develop even with a Higgs boson mass well above the current direct limit of 114 GeV. The φ^6 term can be generated for instance by strong dynamics at the TeV scale or by integrating out heavy particles like an additional singlet scalar field. We discuss conditions to comply with electroweak precision constraints, and point out how future experimental measurements of the Higgs self couplings could test the idea.

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Baryogenesis and the Standard Model: The observed large baryon asymmetry requires natural law to obey three principles: baryon number violation, C and CP violation, and out-of-equilibrium dynamics [1]. In the Standard Model (SM), baryon number violation can occur through the electroweak sphaleron [2, 3], which is a non-perturbative saddle-point solution to the field equations attainable at high temperatures. These solutions allow transitions to topologically distinct $SU(2)$ vacua with differing baryon number.

C is already violated in the SM as well as CP, as evidenced in the Kaon and B-meson systems. Nevertheless, it has been thought [4] that CP violation from the Kobayashi–Maskawa phase is too suppressed to play a dominant role in baryogenesis, although a recent work [5] suggests a way to circumvent this common view. We note also that higher dimensional operators could well provide the desired CP violation [6].

In this letter, we focus on the last main challenge for the viability of SM baryogenesis [7]: the requirement of out-of-equilibrium dynamics. This would be present in the SM if there was a strong first order EWPT. In this case, bubbles of the non-zero Higgs field vev nucleate from the symmetric vacuum and as they expand, particles in the plasma interact with the phase interface in a CP-violating way. The CP asymmetry is converted into a baryon asymmetry by sphalerons in the symmetric phase in front of the bubble wall [8]. One of the strongest constraints on EW baryogenesis comes from the requirement that baryons produced at the bubble wall are not washed out by sphaleron processes after they enter the broken phase. Imposing that sphaleron processes are sufficiently suppressed in the broken phase at the critical temperature leads to the constraint $\langle\varphi(T_c)\rangle/T_c \gtrsim 1$. This bound is very stable with respect to modifications of either the particle physics or of the cosmological evolution as was reviewed in [9]. In the SM, the EWPT is first order if $m_H < 72$ GeV [10] and to suppress sphaleron processes in the broken phase would actually require $m_H \lesssim 35$ GeV. However, the current limit on the Higgs boson mass is well above that at $m_H > 114\,\text{GeV}$ [11], and the SM fails to be an adequate theory for baryogenesis.

As the hopes for a SM solution to baryogenesis faded other ideas have been pursued [12]. One of the most promising ideas presented in the last decade is from supersymmetry. If the superpartner to the top quark is lighter than about 150 GeV, a first-order EWPT can be induced from large-enough cubic interactions in the Higgs potential. This scenario is getting a thorough test as searches for the light top superpartner are rapidly closing the viable parameter space for this solution [13]. Recent ideas to extend the particle spectrum may help resurrect electroweak baryogenesis in supersymmetry [14].

Low-scale cutoff theory: In this work, we focus on a single Higgs doublet model and we study how the dynamics of the EWPT can be affected by modifying the SM Higgs self-interactions. In contrast with previous approaches initiated by ref. [15], we do not rely on large cubic Higgs interactions. Instead, we allow the possibility of a negative quartic coupling while the stability of the potential is restored by higher dimensional operators. We add a φ^6 non-renormalizable operator to the SM potential, and show that it can induce a strong first-order phase transition sufficient to drive baryogenesis [16]. We have numerically checked that adding higher order terms in the potential suppressed by the same cutoff scale will give corrections of a few percent at most to the ratio $\langle\varphi(T_c)\rangle/T_c$ that we computed analytically while restricting ourselves to operators of dimension six or less.

The most general potential of degree six can be written, up to an irrelevant constant term, as

$$V(\Phi) = \lambda \left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 + \frac{1}{\Lambda^2}\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^3 \tag{1}$$

where Φ is the SM electroweak Higgs doublet. At zero

temperature the CP-even scalar state can be expanded in terms of its zero-temperature vacuum expectation value $\langle\varphi\rangle = v_0 \simeq 246\,\text{GeV}$ and the physical Higgs boson H: $\Phi = \varphi/\sqrt{2} = (H + v_0)/\sqrt{2}$.

At zero temperature we can minimize eq. (1) to find λ and v in terms of physical parameters m_H and v_0. We find two possibilities

case	m_H^2	v^2	λ
1	any m_H	v_0^2	$\frac{m_H^2}{2v_0^2} > 0$
2	$m_H^2 < \frac{3}{2}\frac{v_0^4}{\Lambda^2}$	$v_0^2\left(1 - \frac{2\Lambda^2 m_H^2}{3v_0^4}\right)$	$-\frac{m_H^2}{2v_0^2} < 0$

Note that, up to an irrelevant constant, the potential is unchanged by the parameter transformation: $\lambda \to -\lambda$ and $v^2 \to v^2(1 - 4\Lambda^2\lambda/(3v^2))$. So, Case 2 is actually physically equivalent to Case 1. And in the rest of the paper, we restrict ourselves to $\lambda > 0$. $\varphi = v_0$ is the global minimum of the potential as long as $m_H^2 > v_0^4/\Lambda^2$, otherwise $\varphi = 0$ is a deeper minimum. The dynamics of the EWPT depends only on the values of m_H and Λ. It should be noted that, for the region of the parameter space where a first order EWPT occurs, the value of the quartic coupling at the origin will be negative, in contrast to the SM scenario.

We approximate finite temperature effects by adding a thermal mass to the potential $V(\varphi, T) = cT^2\varphi^2/2 + V(\varphi, 0)$, where c is generated by the quadratic terms ($T^2 m_i^2$ where i denote all particles that acquire a φ-dependent mass) in the high-T expansion of the one-loop thermal potential

$$c = \frac{1}{16}\left(4y_t^2 + 3g^2 + g'^2 + 4\frac{m_H^2}{v_0^2} - 12\frac{v_0^2}{\Lambda^2}\right), \qquad (2)$$

g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, and y_t is the top Yukawa coupling. The $T^2 m_i^2$ terms also generate a T-dependent contribution to the Higgs quartic coupling of the form $T^2\varphi^4/(4\Lambda^2)$. In the following, we have discarded this contribution to keep our analytical study simple. We have checked that it does not alter our results by more than a few percent in the physically interesting region where a strongly first-order EWPT occurs. There is also a cubic Higgs interaction induced by finite temperature effects (crucial in supersymmetric baryogenesis) but it has a smaller role in our discussion, and should tend to make the EWPT slightly stronger first-order. While perturbation theory is expected to break down at high temperature, its validity has been confirmed by lattice calculations in the regime where the EWPT is strongly first-order, in the SM [10] as well as in its supersymmetric extension [17]. We therefore expect that the value of $\langle\varphi(T_c)\rangle/T_c$ given by our naive tree level analysis be not too different from its actual value.

The critical temperature T_c at which the minimum at $\varphi \neq 0$ is degenerate with that at $\varphi = 0$ is

$$T_c^2 = \frac{\Lambda^4 m_H^4 + 2\Lambda^2 m_H^2 v_0^4 - 3v_0^8}{16c\Lambda^2 v_0^4}. \qquad (3)$$

The vacuum expectation value of the Higgs field at the critical temperature in terms of m_H, Λ and v_0 is

$$\langle\varphi^2(T_c)\rangle = v_c^2 = \frac{3}{2}v_0^2 - \frac{m_H^2\Lambda^2}{2v_0^2}. \qquad (4)$$

We can see from eqs. (3) and (4) that for any given m_H there is an upper bound on Λ to make sure that the phase transition is first order (i.e. $v_c^2 > 0$), and there is a lower bound on Λ to make sure that the $T = 0$ minimum at $\varphi \neq 0$ is a global minimum (i.e. $T_c^2 > 0$). These two combine to give the important equation

$$\max\left(\frac{v_0^2}{m_H}, \frac{\sqrt{3}v_0^2}{\sqrt{m_H^2 + 2m_c^2}}\right) < \Lambda < \sqrt{3}\frac{v_0^2}{m_H} \qquad (5)$$

where $m_c = v_0\sqrt{(4y_t^2 + 3g^2 + g'^2)/8} \approx 200$ GeV. Note that the coefficient c in the thermal mass is positive if and only if $\Lambda > \sqrt{3}v_0^2/\sqrt{m_H^2 + 2m_c^2}$. Thus when Λ saturates the lower bound in eq. (5), the critical temperature is either vanishing or infinite when m_H is smaller or bigger than m_c respectively. At $m_H = m_c$ and $\Lambda = v_0^2/m_c$, the critical temperature is not uniquely defined but this is an artifact of our approximations. Around that point higher order terms in the thermal potential, like the $T^2\varphi^4/(4\Lambda^2)$ terms or $T\varphi^3$ terms mentioned earlier, will resolve the singularity. These higher order terms will in particular give corrections to the bounds (5) delineating the first order phase transition region.

Figs. 1 and 2 plot contours of constant T_c and v_c/T_c, respectively, in the Λ vs. m_H plane. These results are encouraging and motivate a full one-loop computation of the thermal potential. Such an analysis is underway.

Sphaleron solution: We compute the sphaleron solution of this effective field theory, eq. (1), by starting with the ansatz [3]

$$W_i^a\sigma^a dx^i = -\frac{2i}{g}f(\xi)dU\,U^{-1}, \quad \phi = \frac{v_0}{\sqrt{2}}h(\xi)U\begin{pmatrix}0\\1\end{pmatrix},$$

where $\xi = gv_0 r$ and $U = \frac{1}{r}\begin{pmatrix} z & x+iy \\ -x+iy & z\end{pmatrix}$ and, as usual, $v_0 \simeq 246\,\text{GeV}$. We compute only the $SU(2)$ sphaleron as corrections from $U(1)_Y$ are expected to be small [3, 18]. The functions f and h are solutions to two coupled nonlinear differential equations:

$$\begin{aligned}
\xi^2\frac{d^2 f}{d\xi^2} &= 2f(1-f)(1-2f) - \frac{\xi^2}{4}h^2(1-f) \\
\frac{d}{d\xi}\left[\xi^2\frac{dh}{d\xi}\right] &= 2h(1-f)^2 + \frac{\lambda}{g^2}\xi^2(h^2-1)h \\
&\quad + \frac{3}{4}\frac{v_0^2}{g^2\Lambda^2}\xi^2 h(h^2-1)^2
\end{aligned}$$

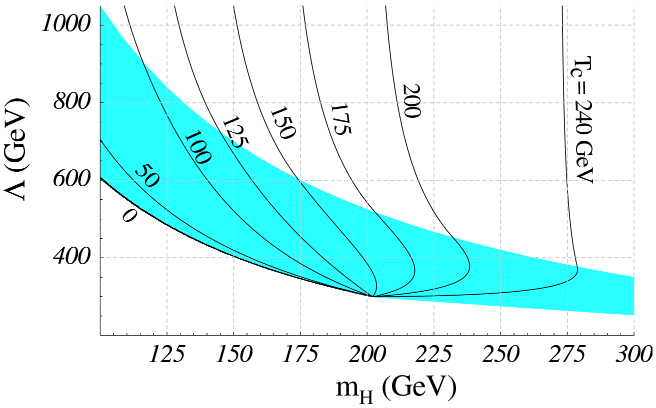

FIG. 1: Contours of constant T_c from 0 to 240 GeV. The shaded blue region satisfies the bounds of eq. (5). Above it, the EWPT is second order and the critical temperature is no more given by eq. (3) but instead by $T_c^2 = (2\Lambda^2 m_H^2 - 3v_0^4)/4c\Lambda^2$.

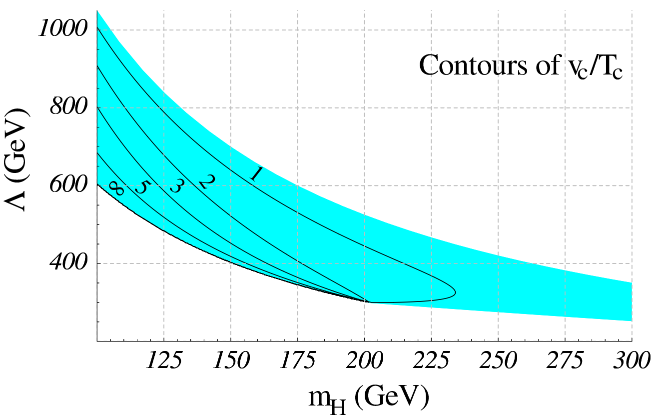

FIG. 2: Contours of constant v_c/T_c from 1 to ∞. The shaded blue region satisfies the bounds of eq. (5).

subject to the boundary conditions $f(0) = h(0) = 0$ and $f(\infty) = h(\infty) = 1$. To solve these differential equations it is necessary to first expand the solutions about their asymptotic values as $\xi \to 0$ and $\xi \to \infty$:

$$\begin{aligned} f(\xi \to 0) &= \xi^2/a_0^2 \quad , \quad h(\xi \to 0) = \xi/b_0 \\ f(\xi \to \infty) &= 1 - a_\infty \exp(-\xi/2) \\ h(\xi \to \infty) &= 1 - (b_\infty/\xi)\exp(-\sigma\xi) \end{aligned}$$

where $\sigma \equiv \sqrt{2\lambda/g^2}$ and a_0, a_∞, b_0 and b_∞ are constants to be determined.

Solving for the constants $(a_0, b_0, a_\infty, b_\infty)$ is equivalent to solving the differential equations and the boundary conditions. We solve them by choosing random numerical values for a_0, a_∞, b_0 and b_∞, shooting the solution from $\xi = \infty$ down to some intermediate $\xi = \xi_{\rm fit}$ and also from $\xi = 0$ up to $\xi = \xi_{\rm fit}$. If both equations match at $\xi_{\rm fit}$ a solution has been found. In practice, we set up a χ^2 fit to measure goodness of match, and require that h, f and their derivatives match to better than one part per million before declaring that a solution has been obtained. This procedure is computer intensive.

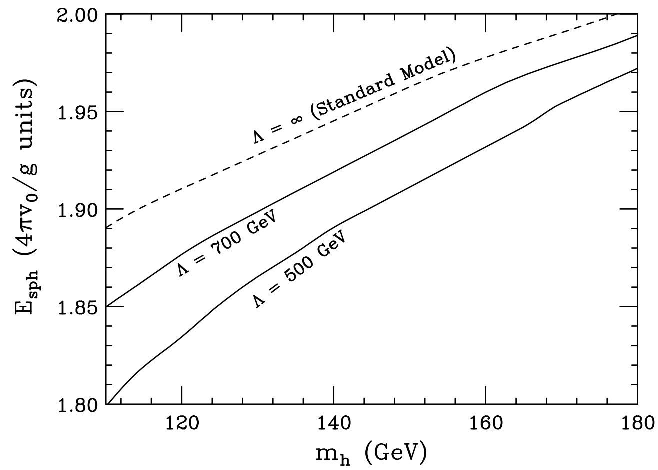

FIG. 3: Sphaleron energy at zero temperature in units of $4\pi v_0/g = 4.75$ TeV.

After obtaining the sphaleron solution we compute the sphaleron energy at $T = 0$ (shown in Fig. 3) according to the equation

$$E_{\rm sph} = \frac{4\pi v_0}{g}\int_0^\infty d\xi \left(4f'^2 + \frac{8}{\xi^2}f^2(1-f)^2 + \frac{1}{2}\xi^2 h'^2 + h^2(1-f)^2 + \frac{\lambda}{4g^2}\xi^2(h^2-1)^2 + \frac{v_0^2}{8g^2\Lambda^2}\xi^2(h^2-1)^3\right) \quad (6)$$

It differs from the SM value by the last term, which tends to make the sphaleron energy slightly smaller (by only a few percent). A similar conclusion was also reached in the MSSM [19].

The sphaleron energy is a crucial quantity for EW baryogenesis as the rate of baryon number violation in the broken phase at T_c is proportional to $e^{-E_{\rm sph}(T_c)/T_c}$ [20]. $E_{\rm sph}(T_c)$ is approximately given by eq. (6) where v_0 is replaced by v_c, and this is how requiring that sphaleron processes be frozen leads to the bound $v_c/T_c \gtrsim 1$. Knowing whether the right-hand side of this inequality is 1 or 1.5 is crucial in deriving the resulting bound on the Higgs mass, and this depends, among other things, on the precise sphaleron energy. The fact that $E_{\rm sph}$ is larger than the cutoff scale for a first-order phase transition is not inconsistent with the calculation of the rate of baryon number violation at T_c. Indeed, $E_{\rm sph}$ is large because the sphaleron is an extended object, but its local energy density is always smaller than the cutoff scale. While a large amount of energy has to be pumped into the thermal bath to build a sphaleron configuration, this does not involve any local physics beyond the cutoff scale.

Precision electroweak constraints: The theory we have presented above is the SM with a low-scale cutoff. It

is minimal in that no new particles have been introduced to achieve the desired out-of-equilibrium first-order phase transition needed for baryogenesis. However, this does not mean that the phenomenology of this model is indistinguishable from that of the SM.

The non-renormalizable operators of this theory can significantly affect observables. If the only additional terms are those given by eq. (1), there would be no phenomenological constraints on this scenario to worry about. However, a low-scale cutoff for other dimension-six operators can be problematic for precision electroweak observables [21]. As an example, let us consider the following four dimension-six operators suppressed by the cutoff scale Λ:

$$\begin{aligned}\Delta\mathcal{L} \;=\; & \frac{\epsilon_\Phi}{\Lambda^2}(\Phi^\dagger D_\mu \Phi)^2 + \frac{\epsilon_W}{\Lambda^2}(D_\rho W^a_{\mu\nu})^2 + \frac{\epsilon_B}{\Lambda^2}(\partial_\rho B_{\mu\nu})^2 \\ & + \frac{\epsilon_F}{\Lambda^2}\bar{\nu}_\mu \gamma_\alpha P_L \mu \bar{e}\gamma^\alpha P_L \nu_e. \end{aligned} \tag{7}$$

The most sensitive precision electroweak observables are $\sin^2\theta_W^{\rm eff}$, m_W, $\Gamma_l = \Gamma(Z \to l^+l^-)$, and Γ_Z. The percent shifts to these observables $\Delta\mathcal{O}_i = \{\sin^2\theta_W^{\rm eff}, m_W({\rm GeV}), \Gamma_l({\rm MeV}), \Gamma_Z({\rm GeV})\}$ induced by $\Delta\mathcal{L}$ are

$$\% \left(\frac{\Delta\mathcal{O}}{\mathcal{O}}\right)_i = \begin{pmatrix} 8.57 & 6.19 & -1.47 & 4.29 \\ -4.31 & -0.55 & -0.55 & -0.65 \\ -7.20 & 1.69 & 0.93 & -3.61 \\ -7.90 & 1.00 & 1.08 & -3.93 \end{pmatrix} \begin{pmatrix} \tilde{\epsilon}_\Phi \\ \tilde{\epsilon}_W \\ \tilde{\epsilon}_B \\ \tilde{\epsilon}_F \end{pmatrix}$$

where $\tilde{\epsilon}_i = \epsilon_i\,(1\,{\rm TeV})^2/\Lambda^2$. The experimental measurements of these observables are [11]

$$\begin{aligned} \sin^2\theta_W^{\rm eff} &= 0.23150 \pm 0.00016 \\ m_W({\rm GeV}) &= 80.426 \pm 0.034 \\ \Gamma_l({\rm MeV}) &= 83.984 \pm 0.086 \\ \Gamma_Z({\rm GeV}) &= 2.4952 \pm 0.0023 \end{aligned}$$

We can compare the experimental values of the observables with the dimension-six operator shifts induced by the cutoff scale Λ. The $(\Phi^\dagger D_\mu \Phi)^2$ operator appears to have the most substantial effect on the precision electroweak observables. This operator is a pure isospin breaking operator and is equivalent to a positive shift in the T parameter in the Peskin–Takeuchi framework ($T \simeq -7.8\,\tilde{\epsilon}_\Phi$).

Barring some nontrivial cancellations of multiple ϵ_i contributions to the precision electroweak observables, it appears that $\epsilon_\Phi \lesssim 10^{-2}$ is necessary if $\Lambda \lesssim 1\,{\rm TeV}$. The other ϵ_i values are somewhat less constrained, but likely need to be nearly as small also. Therefore, if this framework is to be viable there must be a small hierarchy between the $1/\Lambda^2$ coefficient of eq. (1) and the ϵ_i/Λ^2 coefficients of eq. (7). In the absence of a UV completion of the theory, this little hierarchy of high-dimensional operators remains unexplained. We note in passing that the operators can have substantially different conformal weights if the theory at the cutoff is a strongly coupled theory where each field gets large anomalous dimensions. Perhaps this distinguishing property of the operators is a key to the needed hierarchy.

As a concrete example of a possible origin of the non-renormalizable Higgs self-interaction, we note that a $|\Phi|^6$ term can be generated by decoupling a massive degree of freedom. For instance, in a manner similar to ref. [14] we can consider a scalar singlet ϕ_s coupled to the Higgs via

$$\Delta V = \frac{1}{2}m_s^2\phi_s^2 + m\phi_s\Phi^\dagger\Phi + \frac{1}{2}a\phi_s^2\Phi^\dagger\Phi. \tag{8}$$

Assuming that the mass of the singlet is higher than the weak scale, integrating out this scalar degree of freedom gives rise to the additional Higgs interactions (we discard for simplicity other terms, like $M\phi_s^3$ or $\beta\phi_s^4$, that could also be added, since they generically will not change the conclusions):

$$V_{\rm new} = -\frac{m^2}{2m_s^2}|\Phi|^4 + \frac{am^2}{2m_s^4}|\Phi|^6 + \mathcal{O}\left(\frac{a^2m^4|\Phi|^8}{m_s^6}\right). \tag{9}$$

We assume that m and m_s are of the same order to be able to neglect the higher-order terms in the expansion. Therefore, if the mass scale in the singlet sector is around a TeV a ϕ^6 term as well as a negative ϕ^4 term are generated in the Higgs potential. A small fine-tuning between the singlet-induced negative quartic coupling of order 1 and the initial positive quartic coupling of the Higgs potential would be needed to produce a total quartic coupling of order ~ -0.1, as is required to be sitting in the desired region of the (m_H, Λ) plane. Meanwhile, the custodial invariant interactions of eq. (8) will not lead to any of the dangerous operators eq. (7).

Higgs self-couplings as test: Future colliders have the opportunity to test this idea directly by experimentally probing the Higgs potential. When a low-scale cutoff theory alters the Higgs potential with non-renormalizable operators, those same operators will contribute to a shift in the Higgs self-couplings. Expanding around the potential minimum at zero temperature we can find the physical Higgs boson self couplings ($\mathcal{L} = m_H^2H^2/2 + \mu H^3/3! + \eta H^4/4! + \cdots$)

$$\mu = 3\frac{m_H^2}{v_0} + 6\frac{v_0^3}{\Lambda^2}, \quad \eta = 3\frac{m_H^2}{v_0^2} + 36\frac{v_0^2}{\Lambda^2}. \tag{10}$$

The SM couplings are recovered as $\Lambda \to \infty$. In Fig. 4 we plot contours of $\mu/\mu_{\rm SM} - 1$ in the Λ vs. m_H plane.

No experiment to date has meaningful bounds on the H^3 coupling. It is estimated that for the Higgs masses in the range needed for the first-order phase transition presented above, a measurement of the H^3 coupling could be done to within a factor of one at the LHC at $\sqrt{s} = 14\,{\rm TeV}$ with $300\,{\rm fb}^{-1}$ integrated luminosity [22]. This

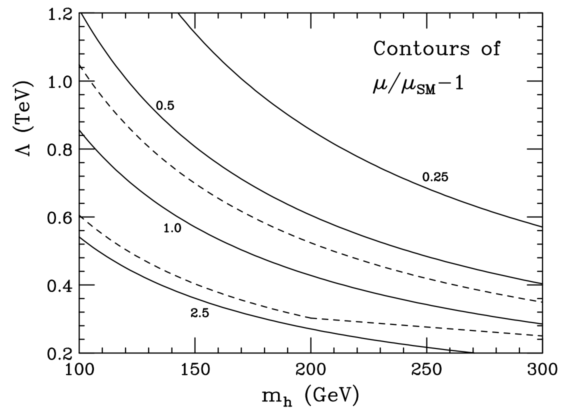

FIG. 4: Contours of constant $\mu/\mu_{\rm SM} - 1$ in the Λ vs. m_H plane. The dashed lines delimit the allowed region defined in eq. (5).

constraint or measurement would be an interesting one for our scenario since a deviation by more than a factor of unity is possible.

In the more distant future, a linear collider at $\sqrt{s} = 500\,\mathrm{GeV}$ and $1\,\mathrm{ab}^{-1}$ of integrated luminosity should be able to measure the coupling to within about 20% [23], and a higher energy linear collider, such as CLIC with $\sqrt{s} = 3\,\mathrm{TeV}$ and $5\,\mathrm{ab}^{-1}$ integrated luminosity, should be able to measure the self-coupling to within a few percent [24]. A few-percent measurement may also be possible at the VLHC at $\sqrt{s} = 200\,\mathrm{TeV}$ with $300\,\mathrm{fb}^{-1}$ integrated luminosity [22].

Conclusion: We have shown that a strong first-order electroweak phase transition is possible within the SM when we take into consideration the effects of a φ^6 Higgs operator with a low cutoff. Higgs masses well above the 114 GeV direct limit are possible within this framework. The main experimental test of this idea is the altered Higgs cubic self-coupling. The LHC should be able to probe $\mathcal{O}(1)$ corrections, but a high-energy linear collider will likely be required to measure the deviation at the tens of percent level accurately.

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